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# Stochastic Stability In Evolutionary Game Theory

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# Game Theory Intro

## Bertrand Russell's Game of "Chicken"

On a deserted road, two cars race towards each other

He who swerves is "chicken"

Model as a game:

Jim's Payoff ($J_{lm}$)

| | | Buzz | |
|---|---|---|---|
| | |  |  |
| Jim |  | -1 | 2 |
| |  | 0 | 1 |

Buzz's Payoff ($B_{lm}$)

| | | Buzz | |
|---|---|---|---|
| | |  |  |
| Jim |  | -1 | 0 |
| |  | 2 | 1 |

2 players, each with 2 pure strategies

1) don't swerve     2) swerve 

# Nash Equilibrium

A choice of strategies by all players is a Nash equilibrium iff for each player, if all of the other players' strategies are held fixed, that player cannot increase his own payoff by changing only his own strategy.

In "Chicken", there are two pure-strategy Nash equilibria:

(Jim doesn't swerve , Buzz swerves )
(Jim swerves , Buzz doesn't swerve )

# Mixed Strategies

A player $X$ may move by rolling a die. If $X$ has $d_X + 1$ pure strategies, a mixed strategy of $X$ is a $(d_X + 1)$-tuple, $(x_0, x_1, \ldots, x_{d_X})$, where $x_i$ is the probability that $X$ chooses $X$'s $i$th pure strategy. So $0 \leq x_i \leq 1$, and $\sum_{i=0}^{d_X} x_i = 1$. The mixed strategies of $X$ are points of the probability simplex.

# Evolutionary Game Theory

*Now we consider a population of organisms. The pure strategies correspond to the various phenotypes. The game is played repeatedly in encounters between pairs of organisms. The payoff to each organism resulting from such an encounter is an increase (or decrease) in fitness. The proportion of each phenotype in the population (which can be considered a mixed strategy of the whole population) evolves according to the dynamical system given by the replicator equation.*

$$\dot{x}_i(t)/x_i(t) = (f_i(x) - \sum_i x_i f_i(x))$$

# Evolutionary Stability and Stochastic Stability

*Evolutionarily Stable Strategy*: Stable against invasion of a different strategy (phenotype).

*Stochastically Stable Strategy*: Stable against continuous small invasions of different strategies.

*Discuss*: Young says it's "biologically unrealistic" that phenotype proportions ever go to zero. Is this realistic?

*Discuss*: Young models whatever goes on besides these pairwise encounters as random noise. Is this the right thing to do?

*Discuss*: Under what conditions will a single stochastically stable pure strategy win out? (Young gives a few examples.)