Programs, Proofs, and Service Networks

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Guiding Analogy

- Type checker for functional programming language yields well-typedness judgements for programs
  - Very special kind of theorem prover
- Theorem prover such as Isabelle/Isar/HOL (`www.cl.cam.uk/Research/HVG/Isabelle`) yields proofs for formulae
  - May be written in strongly typed functional programming language such as Ocaml (`www.ocaml.org`)
- If type system allows *dependent types*, correspondence is exact (Martin-Löf isomorphism between proofs and programs)
  - Basis of Coq and Nuprl theorem provers

Introducing Application to Service Networks

- Will describe service manager by declarations in imaginary functional programming language
  - Similar to ML, but includes dependent types for convenience
  - type α `list n` is a list of n elements of type α
- Service manager uses service providers to construct expressions using services to yield other services
- Service manager proves well-typedness of expressions

Services and Service Types

- `type service`
  - an abstract type
- `type servicelogicformula =`
  `| Opaque of service -> bool`
  `| And of servicelogicformula list`
  - Basic formulae such as $\varphi(x)$ are in more complicated "service logic"
  - Actually have structure, but to service manager are atomic: therefore "opaque"
  - Service is of a type iff it satisfies the corresponding formula $\varphi_1(x) \wedge \varphi_2(x) \wedge \cdots \varphi_n(x)$

Service Providers

```
type serviceprovider n =
(premises: servicelogicformula list n) *
(conclusion: servicelogicformula) *
(useconstraint: service list -> bool) *
(provide:( (resources: service list n)
         * (currentusers: service list) -> service ) ) *
derivation: servicelogicderivation
```

An n-ary service provider with premises $\varphi_1, \ldots, \varphi_n$, conclusion φ, use constraint u and provision function `provide` carries a derivation in the service logic of the formula:

$$\frac{\varphi_1(x_1) \wedge \cdots \wedge \varphi_n(x_n) \wedge u(Y)}{\varphi(\texttt{provide}(x_1, \ldots, x_n, Y))}$$

where the x_i's are services and Y is a list of services of arbitrary length

Semantics of Service Provision

- The x_i's are the *resources* used
- The φ_i's are the *types* of those resources, encoded as formulae in the service logic
- φ is the type of service this service provider provides
- Y is the set of this service provider's current users
- $u(Y)$ is whether the service provider can provide the service to another user despite already serving current users Y
- $f(x_1, \ldots, x_n, Y)$ is the service provided to the newest user

Service Logic Example

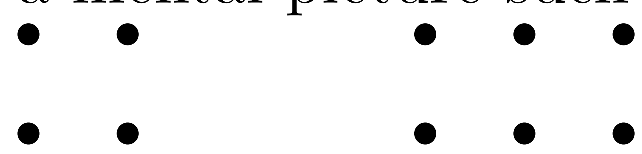
- Let services be integers, and the service logic be elementary number theory
- The service provider `sum` uses as resources two even integers and provides another even integer
  - The use constraint u always evaluates to true
- The service provider proves the formula:

$$\frac{\texttt{even}(n_1) \wedge \texttt{even}(n_2) \wedge \texttt{true}}{\texttt{even}(\texttt{sum}(n_1, n_2))}$$

Semantics of Service Logic Example

Three levels of understanding:

- You who designed the service provider may have had a mental picture such as the following:

  which guided you in creating a proof of the service logic formula
- The service logic oracle knows the Peano axioms for the integers and the definitions of `even` and `sum`, so it can check your proof
- The service manager treats integers, predicates such as `even`, and functions such as `sum` as black boxes

Service Management

```
type servicemanager = blockedserviceprovider list
                    * runningserviceprovider list
```

- Will show mutually recursive type declarations leading up to `blockedserviceprovider` and `runningserviceprovider`
- An n-ary service provider may be *blocked* awaiting some of its resources $x_1, \ldots, x_n$
- When all these are provided, it will start *running* and can provide its resource to others until it is all used up (its use constraint fails)

Proof-related Declarations

```
type goal = servicelogicformula * int
type proof_state n =
  ( resourcepool: runningserviceprovider list ) *
  ( goals: goal list ) *
  ( assignment n: int -> int )
and tactic = proof_state -> proof_state
and derivation n =
  ( tactic_sequence: tactic list ) *
  ( current_goals: goal list ) *
  ( current_assignment n: int -> int )
and...
```

Semantics of Proof-related Declarations

- `goal` (φ, i) stands for $\varphi(x_i)$
- `assignment` maps i with $1 \leq i \leq n$ to j if `runningserviceprovider` j is to provide service x_i, and maps i to -1 if no such `runningserviceprovider` has been found
- `tactic` advances proof state toward easier goals
  - backward tactic works on goals
  - forward tactic works on hypotheses (embedded in running service provider list)
- `derivation` is (almost) a proof state, plus sequence of tactics leading to it

Managed Service Provider Declarations

```
...and blockedserviceprovider n =
  serviceprovider n * derivation n
and serviceuse = runningserviceprovider * service
and runningserviceprovider n =
  serviceprovider n * derivation n true *
  ( resources: serviceuse list n ) *
  ( users: serviceuse list )
```

Completing Semantics of Derivations

- Recall: service provider proves rule

$$\frac{\varphi_1(x_1) \wedge \cdots \wedge \varphi_n(x_n) \wedge u(Y)}{\varphi(f(x_1, \ldots, x_n, Y))}$$

- Initial goal of registered service provider's derivation is rule's premise $\varphi_1(x_1) \wedge \cdots \wedge \varphi_n(x_n)$
- Initial proof state includes `servicemanager`'s `runningserviceprovider` list, assignment always to -1

Managed Service Provider Semantics

- When derivation can make no more progress, blocks
  - Current goals can be seen as pending service requests
- `derivation n true` means assignment maps $1, \ldots, n$ to positive integers—the indices of `resources` in the `runningserviceprovider` list—and current goal list is empty
- if $((p_1, y_1), \ldots, (p_k, y_k))$ is a `runningserviceprovider`'s user list, and u is its use constraint, then it must be the case that $u(y_1, \ldots, \hat{y}_i, \ldots, y_k)$, for any i, i.e., the use constraint is not violated